# MODIVX STANDARD SERIES
# METALOGICAL DIASTAGRAPHY (MLD)

An Axiomatic Framework for the Structural Diagnosis of Formal Expressions

## Mathematical Core (MC) — Version 2.0

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ABSTRACT

MODIVX (Model Diagnostics Verifiable Exchange) introduces an open standard for the structural diagnosis of formal systems and model representations. Its mathematical foundation, Metalogical Diastagraphy (MLD), defines an axiomatic framework for analyzing formal expressions independently of semantic interpretation.

The framework establishes diagnostic dimensions, perturbative structural analysis, adaptive diagnostic trajectories, and meta-adaptive stabilization within a closed formal system.

Rather than reducing diagnostic outcomes to scalar evaluation metrics, MODIVX models diagnosis as a multidimensional structural state space over equivalence classes of formal expressions. Diagnostic behavior is characterized through perturbative reachability, dimensional reactivity, sensitivity, and stability, while adaptive operators provide formally bounded

transformation dynamics guaranteed to stabilize under well-founded progress relations.

The framework is designed for interoperability, verifiable diagnostics, and scalable integration into frontier AI systems, formal reasoning environments, and adaptive computational architectures.

## SCOPE

MLD defines a structural diagnostic framework for formal expressions independently of semantic interpretation.

This document specifies the mathematical core of the MODIVX standard architecture.

The document does not define a runtime implementation, reference implementation, or application-specific instantiation.

## TERMINOLOGICAL CONVENTION

All formally capitalized terms used throughout this document (including but not limited to Diagnostic Dimension, Perturbation Operator, Reactivity, Adaptive Fixpoint, and Meta-Stability) refer to precisely defined notions internal to the MLD framework.

## IMPLEMENTATION NOTICE

MODIVX is implementation-agnostic and does not prescribe a specific runtime, model architecture, execution environment, or optimization strategy.

Conformance to MODIVX requires satisfaction of the formal constraints defined by the corresponding specification layers.

## NON-CLAIM STATEMENT

MLD does not claim semantic truth evaluation, probabilistic correctness estimation, model alignment guarantees, or generalized safety certification.

The framework exclusively formalizes structural diagnostic relations over formal expressions.

## CANONICAL VERSION STATEMENT

This document constitutes the canonical public reference edition of the MODIVX Mathematical Core (MC).

Modified, derived, extended, or forked versions must not be represented as canonical MODIVX specifications without explicit written authorization from the Ruben Jaybird Institute.

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## VERSION HISTORY

v1.0 — Initial Formal Core Release

v2.0 — Extended Reactivity, Adaptive Stabilization, and Meta-Adaptive Formalization

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## OFFICIAL SOURCE

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MODIVX, MLD, and associated canonical specification titles constitute the official reference terminology of the framework.

# METALOGICAL DIASTAGRAPHY (MLD)

Introduction of an Axiomatic Framework for
the Structural Diagnosis of Formal Expressions.
**Mathematical Core (MC) v 2.0.**

Author: Alexander Strohmayer

## INTRODUCTION

This work develops a mathematical framework for the diagnosis of formal expressions. The objects are syntactically structured expressions, considered independently of semantic interpretation. All constructions and results are formulated exclusively in terms of formal structure and relations between such structures.

The framework is organized into six logically dependent blocks (Block I–VI). Each block introduces a necessary component of the diagnostic architecture, and each subsequent block presupposes the constructions of the preceding ones.

Taken together, the blocks define a closed system in which diagnosis, diagnostic behavior, adaptive use of diagnostic results, and the stability of diagnostic systems themselves are formally integrated.

**Formal substrate.** Block I fixes the formal substrate of the framework. Formal expressions are treated up to structural equivalence, induced by admissible structural transformations. All diagnostic notions are defined on equivalence classes rather than on concrete representations, ensuring invariance under representational variation.

Within the same block, admissible perturbation operators, families of structural invariants, and a well-founded ordering on equivalence classes are introduced. This ordering provides the termination basis required for all adaptive constructions developed in later blocks. Without Block I, neither diagnosis nor adaptive processes would be well defined.

**Diagnostic dimensions.** Block II introduces the core notion of diagnostic dimensions. Each diagnostic dimension is associated with a finite, partially ordered state space. Diagnosis assigns to every equivalence class of formal expressions, relative to a purpose context, exactly one state in each diagnostic dimension.

The diagnostic outcome of an expression is therefore a vector of dimension-wise states. No aggregation into a single total order is performed. Block II thus fixes the codomain and structural form of diagnosis and establishes the basis for diagnostic comparability without collapsing distinct diagnostic aspects.

**Determination of diagnostic states.** Block III specifies how diagnostic states are determined. For each diagnostic dimension, a dimension-specific triple is fixed, consisting of a family of invariants, a set of admissible perturbations, and a classification rule.

Diagnostic states are computed from two sources: the invariant satisfaction profile of an expression and the set of equivalence classes reachable through admissible single-step perturbations. Block III thereby renders diagnosis a fully formal, rule-governed assignment derived from explicit structural data.

**Diagnostic behavior.** Block IV lifts diagnosis from isolated states to local diagnostic behavior. Using the perturbation structure fixed earlier, this block defines notions such as reactivity, sensitivity, and stability within each diagnostic dimension.

These notions characterize expressions by the structure of their diagnostic neighborhoods. They do not introduce independent dynamics; rather, they describe how diagnostic states vary under controlled structural change. Block IV thus provides the behavioral layer required for principled use of diagnostic information.

**Diagnostic adaptation.** Block V internalizes diagnosis as a selection principle. Based on the partial orders of diagnostic dimensions, dimension-specific update operators are defined. Each update operator selects an admissible perturbation successor whose diagnostic state is not worse than that of the current expression in the relevant dimension.

Iterated application of these operators yields trajectories of expressions. By construction, and by reliance on the well-founded structure established in Block I, all such trajectories necessarily stabilize. Adaptive transformation is therefore a logical consequence of diagnosis, not an independent primitive of the framework.

**Meta-adaptive closure.** Block VI extends the framework to the level of diagnostic systems themselves. Entire diagnostic schemata—comprising diagnostic dimensions, invariant selections, perturbation spaces, and update operators—are treated as formal objects subject to regulated change.

A corresponding well-founded meta-structure ensures stabilization at this level as well. Block VI does not redefine diagnosis; it closes the framework by internalizing the choice and modification of diagnostic structure as a formally controlled component of the system.

# I. FORMAL SUBSTRATE OF METALOGICAL DIASTAGRAPHY

## 1.1 Formal Expressions

**Definition 1 (Formal Expressions).**

Let $\mathcal{F}$ be a nonempty set. Elements of $\mathcal{F}$ are called formal expressions.

## 1.2 Structural Decomposition

**Definition 2 (Structural Components).**

For each $\varphi \in \mathcal{F}$, the following associated components are given:

1. a symbol inventory $\Sigma(\varphi)$, where $\Sigma(\varphi)$ is a set,
2. an expression structure $\tau(\varphi)$,
3. a core composition $\mu(\varphi)$,
4. a binding structure $\beta(\varphi)$,
5. an interpretation interface $\iota(\varphi)$.

**Axiom 1 (Component Irreducibility).**

For all $\varphi \in \mathcal{F}$, no component among $\Sigma(\varphi), \tau(\varphi), \mu(\varphi), \beta(\varphi), \iota(\varphi)$ is definable as a function of the remaining components alone.

## 1.3 Structural Transformations

**Definition 3 (Structural Transformations).**

Let $\mathcal{T}$ be a set of partial functions $t: \mathcal{F} \rightharpoonup \mathcal{F}$.

Elements of $\mathcal{T}$ are called structural transformations.

**Axiom 2 (Transformation Closure).**

For all $\varphi \in \mathcal{F}$ and all $t \in \mathcal{T}$, if $t(\varphi)$ is defined, then $t(\varphi) \in \mathcal{F}$.

**Axiom 3 (Identity Transformation).**

There exists an element $\mathrm{id} \in \mathcal{T}$ such that for all $\varphi \in \mathcal{F}$, $\mathrm{id}(\varphi) = \varphi$.

**Axiom 4 (Invertibility).**

For every $t \in \mathcal{T}$ there exists $t^{-1} \in \mathcal{T}$ such that for all $\varphi \in \mathcal{F}$ for which both sides are defined, $t^{-1}(t(\varphi)) \equiv \varphi$. If $t(\varphi)$ is defined, then $t^{-1}(t(\varphi))$ is defined and $t^{-1}(t(\varphi)) = \varphi$

**Axiom 5 (Transformation Composition).**

For all $t_1, t_2 \in \mathcal{T}$, the partial composition $t_2 \circ t_1$ belongs to $\mathcal{T}$.

## 1.4 Structural Equivalence

**Definition 4 (Structural Equivalence).**

Define a binary relation $\equiv$ on $\mathcal{F}$ by $\varphi \equiv \psi \Leftrightarrow \exists t \in \mathcal{T}$ such that $t(\varphi) = \psi$.

**Proposition 1 (Equivalence Relation).**

The relation $\equiv$ is reflexive, symmetric, and transitive.

Proof.

Reflexivity follows from Axiom 3. Symmetry follows from Axiom 4. Transitivity follows from Axiom 5.

## 1.5 Perturbation Operators

**Definition 5 (Perturbation Operators).**
Let $\mathcal{P}$ be a set of partial functions $p: \mathcal{F} \rightharpoonup \mathcal{F}$ called perturbation operators, closed under precomposition by structural transformations, i.e. for all $p \in \mathcal{P}$ and all $t \in \mathcal{T}$, the partial composition $p \circ t$ belongs to $\mathcal{P}$.

**Axiom 6 (Perturbation Closure).**
For all $\varphi \in \mathcal{F}$ and all $p \in \mathcal{P}$, if $p(\varphi)$ is defined, then $p(\varphi) \in \mathcal{F}$.

**Axiom 7 (Locality of Perturbations).**
For every $p \in \mathcal{P}$ and every $\varphi \in \mathcal{F}$ such that $p(\varphi)$ is defined, $\left|\Sigma(\varphi) \triangle \Sigma\big(p(\varphi)\big)\right| < \infty$.

**Axiom 8 (Perturbative Non-Triviality).**
There exist $\varphi \in \mathcal{F}$ and $p \in \mathcal{P}$ such that $p(\varphi)/\equiv \varphi$.

### 1.6 Invariants

**Definition 6 (Invariant Predicate).**
An invariant is a function $I: \mathcal{F} \to \{0,1\}$ such that for all $\varphi, \psi \in \mathcal{F}$,
$\varphi \equiv \psi \;\Rightarrow\; I(\varphi) = I(\psi)$.

**Definition 7 (Invariant Family).**
Let $\mathcal{I}$ be a subset of the set of all invariant predicates on $\mathcal{F}$.

### 1.7 Purpose Contexts

**Definition 8 (Purpose Contexts).**
Let $\mathcal{Z}$ be a nonempty set. Elements of $\mathcal{Z}$ are called purpose contexts.
No further structure on $\mathcal{Z}$ is assumed.

**Axiom 9 (Context Parameterization).**
Every diagnostic construction on $\mathcal{F}$ is parameterized by exactly one element of $\mathcal{Z}$.

### 1.8 Well-Founded Diagnostic Progress

**Axiom 10 (Well-Foundedness).**
There exists a well-founded relation $\prec$ on $\mathcal{F}/\equiv$ such that for any sequence $(\varphi_n)_{n\in N}$ with $[\varphi_{n+1}] \in \{[p(\psi)] \mid \psi \in [\varphi_n],\, p \in \mathcal{P},\, p(\psi) \text{ defined}\}$, the induced sequence of equivalence classes $[\varphi_0] \succeq [\varphi_1] \succeq [\varphi_2] \succeq \cdots$ is eventually constant.

### 1.9 Substrate Non-Degeneracy

**Axiom 11 (Substrate Non-Degeneracy).**
For every $z \in \mathcal{Z}$ there exist $\varphi, \psi \in \mathcal{F}$ such that $\varphi \equiv \psi \quad \text{and} \quad \varphi \neq \psi$.

### 2.0 Block Closure

**Proposition 2 (Substrate Closure).**
The structure

$$(\mathcal{F}, \mathcal{T}, \mathcal{P}, \mathcal{I}, \mathcal{Z}, \equiv)$$

constitutes a closed formal substrate for subsequent diagnostic constructions.
Proof.
Immediate from Definitions 1–8 and Axioms 1–11.

## II. DIAGNOSTIC STATE SPACES

### 2.1 Ordinal State Structures

**Definition 9 (Ordinal State Set).**
An ordinal state set is a finite nonempty partially ordered set $(\mathcal{S}, \preceq)$.

### 2.2 Diagnostic Dimensions

**Definition 10 (Dimension Index Set).**
Let $\mathcal{D} = \{1,2,3,4,5,6\}$ be a fixed finite index set. Its elements are called diagnostic dimensions.

### 2.3 Dimension-Specific State Spaces

**Definition 11 (Dimension State Spaces).**
For each $i \in \mathcal{D}$, a pair $(\mathcal{S}_i, \preceq_i)$ is given, where $(\mathcal{S}_i, \preceq_i)$ is an ordinal state set.

**Axiom 12 (Dimensional Non-Triviality).**
For each $i \in \mathcal{D}$, there exist $s_i^-, s_i^+ \in \mathcal{S}_i$ such that $s_i^- \prec_i s_i^+$.

### 2.4 Diagnostic Assignments

**Definition 12 (Diagnostic Assignment System).**
A diagnostic assignment system is a family of functions $(\delta_i)_{i\in\mathcal{D}}$ such that for each $i \in \mathcal{D}$, $\delta_i\colon (\mathcal{F}/\equiv) \times \mathcal{Z} \to \mathcal{S}_i$.
The family $(\delta_i)_{i\in\mathcal{D}}$ is considered fixed data of the diagnostic structure.

**Axiom 13 (Totality of Diagnosis).**
For every $i \in \mathcal{D}$, every equivalence class $[\varphi] \in \mathcal{F}/\equiv$, and every $z \in \mathcal{Z}$, the value $\delta_i([\varphi], z)$ is defined.

### 2.5 Diagnostic State Vectors

**Definition 13 (Diagnostic State Vector).**
Given a diagnostic assignment system $(\delta_i)_{i\in\mathcal{D}}$, the diagnostic state vector of $\varphi \in \mathcal{F}$ under context $z \in \mathcal{Z}$ is

$$\Delta(\varphi, z) \;\coloneqq\; \big(\delta_1([\varphi], z), \dots, \delta_6([\varphi], z)\big) \;\in\; \prod_{i\in\mathcal{D}} c\mathcal{S}_i.$$

This definition is independent of the representative of $[\varphi]$.

### 2.6 Ordering of Diagnostic Vectors

**Definition 14 (Componentwise Diagnostic Order).**
Define a partial order
$\preceq_\Delta$ on $\prod_{i\in\mathcal{D}} cS_i$ by $(s_1, \dots, s_6) \preceq_\Delta (t_1, \dots, t_6) \Leftrightarrow \forall i \in \mathcal{D}\colon s_i \preceq_i t_i$.

**Convention 1 (Non-Aggregation Principle).**
No total order on $\prod_{i\in\mathcal{D}} cS_i$ extending $\preceq_\Delta$ is introduced or assumed.

### 2.7 Boundary States

**Definition 15 (Boundary States).**
Let $i \in \mathcal{D}$. An element $s \in \mathcal{S}_i$ is called a boundary state if

$$(\nexists s' \in \mathcal{S}_i\colon s' \prec_i s) \quad \text{or} \quad (\nexists s' \in \mathcal{S}_i\colon s \prec_i s').$$

**Axiom 14 (Existence of Boundary States).**
For each $i \in \mathcal{D}$, the state space $\mathcal{S}_i$ contains at least one boundary state.

### 2.8 Diagnostic Monotonicity under Perturbation

**Definition 16 (Perturbation Monotonicity).**
Let $i \in \mathcal{D}$. The diagnostic assignment $\delta_i$ is perturbation-monotone if for all $[\varphi] \in \mathcal{F}/{\equiv}, z \in \mathcal{Z}$, and all $p \in \mathcal{P}$ for which $p(\varphi)$ is defined,
$\delta_i([p(\varphi)], z) \preceq_i \delta_i([\varphi], z) \quad \text{or} \quad \delta_i([\varphi], z) \preceq_i \delta_i([p(\varphi)], z).$

**Axiom 15 (Perturbation Comparability).**
For each $i \in \mathcal{D}$, the diagnostic assignment $\delta_i$ is perturbation-monotone.

### 2.9 Interpretative Clarification

**Remark 1 (Diagnostic Decidability).**
Perturbation comparability expresses diagnostic decidability, not semantic relevance. The fact that two diagnostic states are comparable does not imply that the corresponding perturbation is significant, relevant, or informative with respect to the intended interpretation of the dimension.
Equality of diagnostic states represents diagnostic neutrality.

### 3.0 Block Closure

**Proposition 3 (Well-Defined Diagnostic Structure).**
Given the data $((\mathcal{S}_i, \preceq_i)_{i\in\mathcal{D}}, (\delta_i)_{i\in\mathcal{D}})$,
the mapping

$$\Delta\colon \mathcal{F} \times \mathcal{Z} \to \prod_{i\in\mathcal{D}} \mathrm{c}\mathcal{S}_i$$

is well defined and takes values in a partially ordered diagnostic state space.
Proof.
Well-definedness follows from Definitions 11–13 and Axiom 13.
The partial order is given by Definition 14.

## III. DIMENSION-SPECIFIC DETERMINATION LOGICS

### 3.1 General Determination Schema

**Definition 17 (Determination Triple).**
For each diagnostic dimension $i \in \mathcal{D}$, a determination triple $\mathfrak{D}_i := (\mathcal{I}_i, \mathcal{P}_i, \kappa_i)$ is given, where:

- $\mathcal{I}_i \subseteq \mathcal{I}$ is a set of invariant predicates,
- $\mathcal{P}_i \subseteq \mathcal{P}$ is a set of admissible perturbation operators,
- $\kappa_i$ is a classification rule (Definition 21).

### 3.2 Invariant Satisfaction Profiles

**Definition 18 (Invariant Satisfaction Profile).**

Let $i \in \mathcal{D}$. For $[\varphi] \in \mathcal{F}/\equiv$, define the invariant satisfaction profile

$\mathrm{Inv}_i([\varphi]) \colon \mathcal{I}_i \to \{0,1\}, \quad I \mapsto I(\varphi).$

**Proposition 4 (Well-Definedness of Invariant Profiles).**

For all $i \in \mathcal{D}$ and all $[\varphi] \in \mathcal{F}/\equiv$, the map $\mathrm{Inv}_i([\varphi])$ is independent of the representative of $[\varphi]$.

Proof.

By Definition 17, $\mathcal{I}_i \subseteq \mathcal{I}$.

By Definition 6, every $I \in \mathcal{I}$ is invariant under \equiv.

Hence $I(\varphi) = I(\psi)$ whenever $\varphi \equiv \psi$, and the claim follows.

### 3.3 Saturated Perturbation Images

**Definition 19 (Saturated Perturbation Image Set).**

Let $i \in \mathcal{D}$. For $[\varphi] \in \mathcal{F}/\equiv$, define

$$\mathrm{Img}_i([\varphi]) \coloneqq \{[p(\psi)] \in \mathcal{F}/\equiv \mid \psi \in [\varphi],\ p \in \mathcal{P}_i,\ p(\psi) \text{ defined}\}.$$

**Axiom 16 (Perturbative Coverage).**

For each $i \in \mathcal{D}$ and each $[\varphi] \in \mathcal{F}/\equiv$ , $\mathrm{Img}_i([\varphi]) \neq \emptyset$.

**Proposition 5 (Well-Definedness of Saturated Images).**

For all $i \in \mathcal{D}$ and all $[\varphi] \in \mathcal{F}/\equiv$, the set $\mathrm{Img}_i([\varphi])$ depends only on the equivalence class $[\varphi]$.

Proof.

If $[\varphi] = [\varphi']$, then both have the same set of representatives.

The defining comprehension therefore yields the same subset of $\mathcal{F}/\equiv$.

### 3.4 Classification Logic

**Definition 20 (Classification Data).**

Fix $i \in \mathcal{D}$.

For $[\varphi] \in \mathcal{F}/\equiv$ and $z \in \mathcal{Z}$, the classification data is

$(\mathrm{Inv}_i([\varphi]), \mathrm{Img}_i([\varphi]), z) \in \{0,1\}^{\mathcal{I}_i} \times \mathcal{P}(\mathcal{F}/\equiv) \times \mathcal{Z}.$

**Definition 21 (Classification Rule).**

For each $i \in \mathcal{D}$, the map $\kappa_i \colon \left(\{0,1\}^{\mathcal{I}_i} \times \mathcal{P}(\mathcal{F}/\equiv) \times \mathcal{Z}\right) \to \mathcal{S}_i$

is called the classification rule of dimension $i$.

**Axiom 17 (State Assignment).**

For all $i \in \mathcal{D}$, all $[\varphi] \in \mathcal{F}/\equiv$, and all $z \in \mathcal{Z}$, $\delta_i([\varphi], z) = \kappa_i(\mathrm{Inv}_i([\varphi]), \mathrm{Img}_i([\varphi]), z)$.

**Axiom 18 (Determinacy).**

For each $i \in \mathcal{D}$, the diagnostic assignment $\delta_i$ is the unique function satisfying Axiom 17.

### 3.5 Dimensional Non-Triviality and Separation

**Axiom 19 (Non-Triviality of Each Dimension).**

For each $i \in \mathcal{D}$, there exist $[\varphi], [\psi] \in \mathcal{F}/\equiv$ and $z \in \mathcal{Z}$ such that

$\delta_i([\varphi], z) \neq \delta_i([\psi], z).$

**Axiom 20 (Dimensional Separability).**

For each pair $i \neq j$ in $\mathcal{D}$, there exist $[\varphi], [\psi] \in \mathcal{F}/\equiv$ and $z \in \mathcal{Z}$ such that

$\delta_i([\varphi], z) \neq \delta_i([\psi], z)$ and $\delta_j([\varphi], z) = \delta_j([\psi], z)$.

### 3.6 Block Closure

**Proposition 6 (Well-Defined Dimensional Diagnostics).**

For each $i \in \mathcal{D}$, the diagnostic assignment $\delta_i\colon (\mathcal{F}/\equiv) \times \mathcal{Z} \to \mathcal{S}_i$ is well defined and uniquely determined by the triple $\mathfrak{D}_i$.

Proof.

Well-definedness of $\mathrm{Inv}_i$ and $\mathrm{Img}_i$ follows from Propositions 4 and 5.

Existence and uniqueness of $\delta_i$ follows from Axioms 17 and 18.

## IV. REACTIVITY AND SENSITIVITY THEORY

### 4.1 Order-Theoretic Convention

**Definition 22 (Strict Order).**

For each $i \in \mathcal{D}$, define the strict order $\prec_i$ on $\mathcal{S}_i$ by $s \prec_i t \Leftrightarrow (s \preceq_i t \wedge s \neq t)$.

### 4.2 Immediate Diagnostic State Images

**Definition 23 (Immediate State Image).**

Let $i \in \mathcal{D}$. For $[\varphi] \in \mathcal{F}/\equiv$ and $z \in \mathcal{Z}$, define

$$\mathrm{States}_i([\varphi], z) := \big\{\delta_i([\psi], z) \;\big|\; [\psi] \in \mathrm{Img}_i([\varphi])\big\} \subseteq \mathcal{S}_i.$$

### 4.3 Perturbative Reachability

**Definition 24 (Perturbative Reachability).**

Let $i \in \mathcal{D}$. For $[\varphi] \in \mathcal{F}/\equiv$, define

$$\mathrm{Reach}_i([\varphi]) := \bigcup_{n \in N} \mathrm{Img}_i^{(n)}([\varphi]),$$

where:

$$\mathrm{Img}_i^{(0)}([\varphi]) := \{[\varphi]\}, \quad \mathrm{Img}_i^{(n+1)}([\varphi]) := \bigcup_{[\psi] \in \mathrm{Img}_i^{(n)}([\varphi])} \mathrm{r}Img_i([\psi]).$$

**Definition 25 (Reach-State Set).**

Let $i \in \mathcal{D}$. For $[\varphi] \in \mathcal{F}/\equiv$ and $z \in \mathcal{Z}$ , define

$$\mathrm{RStates}_i([\varphi], z) := \{\delta_i([\psi], z) \backslash\mathrm{big}| \; [\psi] \in \mathrm{Reach}_i([\varphi])\} \subseteq \mathcal{S}_i.$$

### 4.4 Reactivity

**Definition 26 (Dimensional Reactivity).**

$[\varphi]$ is reactive in dimension $i$ under context $z$ if

$$\exists [\psi] \in \mathrm{Img}_i([\varphi])\colon \delta_i([\psi], z) \neq \delta_i([\varphi], z).$$

**Definition 27 (Strong Reactivity).**

$[\varphi]$ is strongly reactive in dimension $i$ under context $z$ if

$$\forall [\psi] \in \mathrm{Img}_i([\varphi])\colon \delta_i([\psi], z) \neq \delta_i([\varphi], z).$$

**Definition 28 (Non-Reactivity).**
$[\varphi]$ is non-reactive in dimension $i$ under context $z$ if

$$\forall[\psi] \in \mathrm{Img_i}([\varphi]): \delta_i([\psi], z) = \delta_i([\varphi], z).$$

## 4.5 Sensitivity

**Definition 29 (Dimensional Sensitivity).**
$[\varphi]$ is sensitive in dimension $i$ under context $z$ if $\exists s_1, s_2 \in \mathrm{States_i}([\varphi], z): s_1 \prec_i s_2$.
**Definition 30 (Uniform Sensitivity).**
$[\varphi]$ is uniformly sensitive in dimension $i$ under context $z$ if there exist
$s_{min}, s_{max} \in \mathcal{S}_i$ with $s_{min} \prec_i s_{max}$ such that $\mathrm{States_i}([\varphi], z) = \{s_{min}, s_{max}\}$.

## 4.6 Order-Theoretic Reactivity Order

**Definition 31 (Downward Closure).**
For $A \subseteq \mathcal{S}_i$, define $\downarrow A \coloneqq \{ s \in \mathcal{S}_i \mid \exists a \in A: s \preceq_i a \}$.
**Definition 32 (Reactivity Preorder).**
Fix $i \in \mathcal{D}$ and $z \in \mathcal{Z}$. Define

$$[\varphi] \preceq_i^{\mathrm{react}} [\psi] \Longleftrightarrow \downarrow \mathrm{States_i}([\varphi], z) \subseteq \downarrow \mathrm{States_i}([\psi], z).$$

**Definition 33 (Reactivity Equivalence).**

$$[\varphi] \sim_i^{\mathrm{react}} [\psi] \Leftrightarrow [\varphi] \preceq_i^{\mathrm{react}} [\psi] \wedge [\psi] \preceq_i^{\mathrm{react}} [\varphi].$$

**Proposition 7 (Coherence of the Reactivity Preorder).**
For fixed $i$ and $z$, $\preceq_i^{\mathrm{react}}$ is a preorder and $\sim_i^{\mathrm{react}}$ is the induced equivalence relation.
Proof.
Immediate from reflexivity and transitivity of set inclusion.

## 4.7 Stability

**Definition 34 (Dimensional Stability).**
$[\varphi]$ is stable in dimension $i$ under context $z$ if

$$\forall[\psi] \in \mathrm{Reach_i}([\varphi]): \delta_i([\psi], z) = \delta_i([\varphi], z).$$

**Proposition 8 (Stability Implies Non-Reactivity).**
If $[\varphi]$ is stable in dimension $i$ under context $z$, then it is non-reactive in dimension $i$ under $z$.
Proof.
Since $\mathrm{Img_i}([\varphi]) \subseteq \mathrm{Reach_i}([\varphi])$, stability yields equality on immediate images.

## 4.8 Cross-Dimensional Reactivity

**Definition 35 (Cross-Dimensional Reactivity).**
$[\varphi]$ is cross-reactive under context $z$ if there exist $i \neq j$ such that
$[\varphi]$ is reactive in dimension $i$ and non-reactive in dimension $j$ under $z$.

## 4.9 Interpretative Convention

**Convention 2 (Immediate vs. Transitive Scope).**
Reactivity and Sensitivity are evaluated on immediate images

$$\mathrm{Img}_{\mathrm{i}}(one - step).$$

Stability is evaluated on transitive reachability $\mathrm{Reach}_{\mathrm{i}}$.

### 5.0 Block Closure

**Proposition 9 (Well-Defined Reactivity Theory).**

For each $i$ and $z$, all notions introduced in Definitions 22–35 are well defined on $\mathcal{F}/\equiv$.

Proof.

All constructions depend only on $\mathrm{Img}_{\mathrm{i}}$, $\delta_{\mathrm{i}}$, $\preceq_{\mathrm{i}}$, and set-theoretic operations, each well defined by Blocks I–III.

## V. SELF-ADAPTIVE DIAGNOSTIC OPERATOR

### 5.1 Adaptive Update Structure

**Definition 36 (Adaptive Update Operator).**

For each diagnostic dimension $\mathrm{i} \in \mathcal{D}$, an adaptive update operator is a function

$$\mathcal{A}_i : (\mathcal{F}/\equiv) \times \mathcal{Z} \to \mathcal{F}/\equiv.$$

The family $(\mathcal{A}_i)_{\mathrm{i} \in \mathcal{D}}$ is considered fixed data of the adaptive structure.

**Axiom 21 (Perturbative Realizability).**

For each $\mathrm{i} \in \mathcal{D}$, each $[\varphi] \in \mathcal{F}/\equiv$, and each $\mathrm{z} \in \mathcal{Z}$, $\mathcal{A}_i([\varphi], \mathrm{z}) \in \mathrm{Img}_{\mathrm{i}}([\varphi])$.

### 5.2 Adaptive Progress

**Definition 37 (Adaptive Progress Condition).**

Let $\mathrm{i} \in \mathcal{D}$. The update operator $\mathcal{A}_i$ satisfies the adaptive progress condition if for all $[\varphi] \in \mathcal{F}/\equiv$ andz $\in \mathcal{Z}$, $\delta_{\mathrm{i}}(\mathcal{A}_i([\varphi], \mathrm{z}), \mathrm{z}) \preceq_{\mathrm{i}} \delta_{\mathrm{i}}([\varphi], \mathrm{z})$.

**Axiom 22 (Dimensionwise Progress).**

For each $\mathrm{i} \in \mathcal{D}$ , the update operator $\mathcal{A}_i$ satisfies the adaptive progress condition.

### 5.3 Adaptive Dynamics

**Definition 38 (Adaptive Trajectory).**

Fix $\mathrm{i} \in \mathcal{D}$, $\mathrm{z} \in \mathcal{Z}$, and an initial class $[\varphi_0] \in \mathcal{F}/\equiv$.

Define the adaptive trajectory $([\varphi_{\mathrm{n}}])_{\mathrm{n} \in \mathrm{N}}$ recursively by $[\varphi_{\mathrm{n+1}}] \coloneqq \mathcal{A}_i([\varphi_{\mathrm{n}}], \mathrm{z})$.

### 5.4 Fixpoints and Convergence

**Definition 39 (Adaptive Fixpoint).**

An equivalence class $[\varphi] \in \mathcal{F}/\equiv$ is an adaptive fixpoint in dimension $i$ under context $z$ if $\mathcal{A}_i([\varphi], \mathrm{z}) = [\varphi]$.

**Definition 40 (Adaptive Stabilization).**

An adaptive trajectory $([\varphi_{\mathrm{n}}])_{\mathrm{n} \in \mathrm{N}}$ stabilizes if it is eventually constant, i.e.

$$\exists N \in N\, \forall n \geq N\colon\ [\varphi_n] = [\varphi_N].$$

**Proposition 10 (Stabilization of Adaptive Trajectories).**

For each i $\in \mathcal{D}$, each z $\in \mathcal{Z}$, and each initial $[\varphi_0] \in \mathcal{F}/\equiv$, the adaptive trajectory defined in Definition 38 stabilizes.

Proof.

Define inductively a sequence $(\psi_n)_{n\in N}$ in $\mathcal{F}$ together with a sequence $(p_n)_{n\in N}$ in $\mathcal{P}$ as follows:

Choose $\psi_0 \in \mathcal{F}$ such that $[\psi_0] = [\varphi_0]$. Assume $\psi_n$ chosen with $[\psi_n] = [\varphi_n]$.

By Axiom 21, $[\varphi_{n+1}] = \mathcal{A}_i([\varphi_n], z) \in \mathrm{Img}_i([\varphi_n])$.

By Definition 19, this implies that there exist $\theta_n \in [\varphi_n]$ and $q_n \in \mathcal{P}_i \subseteq \mathcal{P}$ such that $q_n(\theta_n)$ is defined and $[\varphi_{n+1}] = [q_n(\theta_n)]$.

Since $\theta_n \in [\varphi_n] = [\psi_n]$, there exists $t_n \in \mathcal{T}$ with $t_n(\psi_n) = \theta_n$ (Definition 4).

Define $p_n \coloneqq q_n \circ t_n$. By Definition 5, $p_n \in \mathcal{P}$ and $p_n$ is defined at $\psi_n$.

Set $\psi_{n+1} \coloneqq p_n(\psi_n)$. Then $p_n(\psi_n) = q_n\big(t_n(\psi_n)\big) = q_n(\theta_n)$,

hence $[\psi_{n+1}] = [q_n(\theta_n)] = [\varphi_{n+1}]$.

Thus, for all $n$, $\psi_{n+1} = p_n(\psi_n)$ with $p_n \in \mathcal{P}$, and $[\psi_n] = [\varphi_n]$.

By Axiom 10, the sequence of classes $([\psi_n])$ is eventually constant.

Since $[\psi_n] = [\varphi_n]$ for all $n$, the sequence $([\varphi_n])$ is eventually constant.

**Corollary 1 (Existence of Reached Fixpoints).**

Under the hypotheses of Proposition 10, there exists N ∈ N such that $[\varphi_N]$ is an adaptive fixpoint in dimension $i$ under $z$.

Proof.

By Proposition 10, there exists $N$ such that $[\varphi_{n+1}] = [\varphi_n]$ for all $n \geq N$.

But $[\varphi_{N+1}] = \mathcal{A}_i([\varphi_N], z)$ by Definition 38, hence $\mathcal{A}_i([\varphi_N], z) = [\varphi_N]$.

**Definition 41 (Adaptive Stability).**

An equivalence class $[\varphi] \in \mathcal{F}/\equiv$ is adaptively stable in dimension i under context $z$ if the adaptive trajectory starting at $[\varphi]$ reaches an adaptive fixpoint after finitely many steps.

## 5.5 Stationarity under Diagnostic Non-Reactivity

**Axiom 23 (Stationarity on Non-Reactivity).**

Let i $\in \mathcal{D}$. For all $[\varphi] \in \mathcal{F}/\equiv$ and z $\in \mathcal{Z}$, if $[\varphi]$ is non-reactive in dimension $i$ under $z$ (Definition 28), then $\mathcal{A}_i([\varphi], z) = [\varphi]$.

**Proposition 11 (Non-Reactivity Implies Adaptive Fixpoint).**

If $[\varphi]$ is non-reactive in dimension $i$ under context $z$, then $[\varphi]$ is an adaptive fixpoint in dimension $i$ under $z$.

Proof.

Immediate from Axiom 23 and Definition 39.

**Proposition 12 (Diagnostic Stability Implies Adaptive Fixpoint).**

If $[\varphi]$ is stable in dimension $i$ under context $z$ (Definition 34), then $[\varphi]$ is an adaptive fixpoint in dimension $i$ under $z$.

Proof.

By Proposition 8, stability implies non-reactivity. Then Proposition 11 applies.

## 5.6 Global Adaptive Control (Type-Correct)

**Definition 42 (Dimension Selection Rule).**

A dimension selection rule is a function $g: (\mathcal{F}/\equiv) \times \mathcal{Z} \to \mathcal{D}$. The map $g$ is considered fixed data of the adaptive structure.

**Definition 43 (Global Adaptive Operator).**

Given $g$, define $\mathcal{A}([\varphi], z) := \mathcal{A}_{g([\varphi],z)}([\varphi], z)$.

**Definition 44 (Global Adaptive Trajectory).**

Fix $z \in \mathcal{Z}$ and $[\varphi_0] \in \mathcal{F}/\equiv$. Define $([\varphi_n])_{n\in N}$ by $[\varphi_{n+1}] := \mathcal{A}([\varphi_n], z)$.

**Proposition 13 (Stabilization of Global Adaptive Trajectories).**

For each $z \in \mathcal{Z}$ and each initial $[\varphi_0] \in \mathcal{F}/\equiv$, the global adaptive trajectory stabilizes.

Proof.

Proceed as in Proposition 10, but with the index $i_n := g([\varphi_n], z)$ varying with $n$.

Choose $\psi_0 \in \mathcal{F}$ with $[\psi_0] = [\varphi_0]$. Assume $[\psi_n] = [\varphi_n]$.

Then $[\varphi_{n+1}] = \mathcal{A}([\varphi_n], z) = \mathcal{A}_{i_n}([\varphi_n], z) \in \mathrm{Img}_{i_n}([\varphi_n])$ by Axiom 21.

By Definition 19, there exist $\theta_n \in [\varphi_n]$ and $q_n \in \mathcal{P}_{i_n} \subseteq \mathcal{P}$ such that $q_n(\theta_n)$ is defined and $[\varphi_{n+1}] = [q_n(\theta_n)]$.

Choose $t_n \in \mathcal{T}$ with $t_n(\psi_n) = \theta_n$, and set $p_n := q_n \circ t_n$, $\psi_{n+1} := p_n(\psi_n)$.

Then $[\psi_{n+1}] = [\varphi_{n+1}]$ and $\psi_{n+1} = p_n(\psi_n)\text{with}p_n \in \mathcal{P}$.

Apply Axiom 10 to $(\psi_n)$. Then $([\psi_n])$ is eventually constant, hence so is $([\varphi_n])$.

### 5.7 Block Closure

**Proposition 14 (Well-Defined Self-Adaptive System).**

Given the data $(\mathcal{A}_i)_{i\in\mathcal{D}}$ and $g$, the operators $\mathcal{A}$ and the associated trajectories (Definitions 43–44) are well defined and stabilize; moreover, any diagnostically stable class in any dimension is an adaptive fixpoint for that dimension.

Proof.

Well-definedness is immediate from Definitions 36, 42–44.

Stabilization follows from Proposition 13.

The final claim follows from Proposition 12.

## VI. META-ADAPTIVITY

### 6.1 Meta-Adaptation Space

**Definition 45 (Adaptation Schemes)**

Let $\mathfrak{T}$ be a nonempty set whose elements are called adaptation schemes.

Each $\Theta \in \mathfrak{T}$, determines:

a family of diagnostic maps $\{\delta_i^{\Theta}\}_{i\in\mathcal{D}}$,

a family of update operators $\{\mathcal{A}_i^{\Theta}\}_{i\in\mathcal{D}}$,

such that all objects induced by $\Theta$ satisfy the axioms of Blocks II–V.

(No uniqueness or completeness is assumed.)

**Definition 46 (Meta-Observation Space)**

Let $\mathfrak{H}$ be a nonempty set whose elements are called meta-observations.

No internal structure on $\mathfrak{H}$ is assumed.

## 6.2 Meta-Dynamics

**Definition 47 (Meta-Update Relation)**

A meta-update relation is a binary relation $\longrightarrow_{\Theta} \subseteq \mathfrak{T} \times \mathfrak{H} \times \mathfrak{T}$.
We write $\Theta \to \mathrm{h}_{\Theta} \Theta'$ if $(\Theta, \mathrm{h}, \Theta') \in \longrightarrow_{\Theta}$.

**Definition 48 (Meta-Trajectory)**

A meta-trajectory is a sequence $(\Theta_{\mathrm{n}})_{\mathrm{n} \in \mathbb{N}}$
such that for every $n$ there exists h_n $\in \mathfrak{H}$ with $\Theta_{\mathrm{n}} \longrightarrow \mathrm{h}_{\mathrm{n}\,\Theta} \Theta_{\mathrm{n}+1}$.

## 6.3 Meta-Progress Structure

**Axiom 24 (Meta-Progress Ordering)**

There exists a relation $\prec_{\Theta} \subseteq \mathfrak{T} \times \mathfrak{T}$
such that: $\prec_{\Theta}$ is well-founded, for every meta-update $\Theta \longrightarrow \mathrm{h}_{\Theta} \Theta'$,
it holds that $\Theta' \preceq_{\Theta} \Theta$.
(Meta-adaptation cannot strictly increase complexity indefinitely.)

## 6.4 Meta-Stability

**Definition 49 (Meta-Fixpoint)**

An adaptation scheme $\Theta^{*} \in \mathfrak{T}$ is a meta-fixpoint if there exists no $\mathrm{h} \in \mathfrak{H}$ and no $\Theta' \neq \Theta^{*}$ such that $\Theta^{*} \longrightarrow \mathrm{h}_{\Theta} \Theta'$.

**Definition 50 (Meta-Stability)**

A meta-trajectory $(\Theta_{\mathrm{n}})$ is meta-stable if it is eventually constant.

## 6.5 Meta-Stabilization Result

**Proposition 15 (Meta-Stabilization)**

Every meta-trajectory is meta-stable.
Proof.
By Axiom 24, the sequence $(\Theta_{\mathrm{n}})$ is non-increasing with respect to the well-founded relation $\prec_{\Theta}$. Hence it must eventually be constant.

## 6.6 Block Closure

Block VI introduces:

- a genuine second-order state space $\mathfrak{T}$,
- a non-functional meta-dynamics,
- an independent meta-progress structure,
- and a non-trivial stabilization theorem.